

A comment on causality.

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ABSTRACT

We start from the well-known form of the interval of the special relativity, stare it, and build up an attempt to implement the causality from it. Some features appear to be new, they involve the mass of the particle and the structure of space-time.

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Causality supports the construction of several modern theories in physics [1]. Its interests abroad and overcome the very beginning of special relativity and goes to quantum mechanics at S-matrix level when the unitarity condition

$$S^\dagger S = 1$$

does not means causality, rather it just means time development which conserves probability, if we want to incorporate a causal description we have to deal with the chronological order, but this is also a weak condition. Another manner of state causality independent of special relativity and quantum mechanics is via the concept of locality, i.e., all interactions occur at a point so there is no action at a distance, and this is used by relativistic quantum field theory. As an example, in the context of quantum field theories, the choice of the light cone gauge leads in pathological non physical problems. The origin of these problems lays in considerations of non causal distributions and they disappear when causality is restored [2]. There is also defined the microscopic causality as the idea, based on special relativity, in which information cannot travel faster than the speed of light and this is used to demonstrate the spin statistics theorem.

We are not defying the other previous ideas of implement causality in a theory, we just wonder to have an approximation from another perspective to deem causality. Our analyze is as simple as possible, so we are not going into quantum theory neither general relativity. Besides we are just considering the theory of special relativity as our arena and we are going to make use of de Sitter spacetime, so in the first section we will summarize the de Sitter and Anti de Sitter spacetimes. The next section is devoted to explain our form of implement causality.

1 Review of (Anti)de Sitter spacetime.

In this section we will make a briefly review of Anti de Sitter and de Sitter spacetime in four dimensions, that belong to the set of constant curvature spacetime metrics. This kind of metrics are

locally characterized by the condition

$$R_{abcd} = \frac{1}{12}R(g_{ac}g_{bd} - g_{ad}g_{bc}),$$

where the indexes runs over $0, \dots, 3$. This equation is equivalent to $C_{ab} = 0 = R_{ab} - \frac{1}{4}Rg_{ab}$, and this implies that the Riemann tensor is completely determined by the Ricci scalar R . Using Bianchi identities we can conclude that R is constant over all spacetime. In this way we can also classified this spacetime according the sign of R as

$R < 0$ Anti de Sitter spacetime,

$R = 0$ Minkowski spacetime,

$R > 0$ de Sitter spacetime.

1.1 De Sitter spacetime.

Here we will brusquely describe the above case for $R > 0$ i.e., the de Sitter spacetime, which has the R^1xS^3 topology. It can be visualized as the hyperboloid

$$-v^2 + w^2 + x'^2 + y'^2 + z'^2 = \alpha^2$$

in a five dimensional space R^5 with metric

$$-dv^2 + dw^2 + dx'^2 + dy'^2 + dz'^2 = ds^2. \quad (1)$$

Introducing the coordinates $(\hat{t}, \xi, \theta, \phi)$ on the hyperboloid with the relations

$$\begin{aligned} \alpha \sinh(t/\alpha) &= v, \\ \alpha \cosh(t/\alpha) \cos \xi &= w, \\ \alpha \cosh(t/\alpha) \sin \xi \cos \theta &= x', \\ \alpha \cosh(t/\alpha) \sin \xi \sin \theta \cos \phi &= y', \\ \alpha \cosh(t/\alpha) \sin \xi \sin \theta \sin \phi &= z', \end{aligned}$$

using these coordinates, the metric takes the following form

$$ds^2 = -d\hat{t}^2 + \alpha^2 \cosh^2(t/\alpha) [d\xi^2 + \sin^2(\xi)(d\theta^2 + \sin^2\theta d\phi^2)],$$

and this is called the spherical coordinates. There is another interesting form of the metric named planar coordinates in which

$$\begin{aligned} t &= \alpha \ln \frac{v+w}{\alpha}, \\ x &= \alpha \frac{x'}{v+w}, \\ y &= \alpha \frac{y'}{v+w}, \\ z &= \alpha \frac{z'}{v+w}, \end{aligned}$$

then, the metric is

$$-dt^2 + e^{2t/\alpha} \cdot (dx^2 + dy^2 + dz^2) \doteq d\mathcal{S}^2,$$

We can also be able to see that sections of constant t'' are spheres S^3 with positive curvature, where we define the temporal coordinate t'' as

$$t'' = 2 \arctan(e^{t'/\alpha}) - \frac{\pi}{2}, \quad -\frac{\pi}{2} < t'' < \frac{\pi}{2}.$$

then the metric turns to be

$$ds^2 = \alpha^2 \cosh^2(t'/\alpha) [-dt''^2 + d\xi^2 + \sin^2 \xi (d\theta^2 + \sin^2 \theta d\phi^2)].$$

This shows us that the de Sitter spacetime is conformal to the Einstein's static universe which is defined by the metric

$$ds^2 = -dt''^2 + dr'^2 + \sin^2 r' (d\theta^2 + \sin^2 \theta d\phi^2).$$

This describes a universe of positive cosmological constant $\Lambda > 0$.

1.2 Anti de Sitter spacetime.

The space of constant negative curvature, $R < 0$, is denominated as Anti de Sitter spacetime. It has the $S^1 \times R^3$ topology and can be represented as being the hyperboloid

$$-u^2 - v^2 + x^2 + y^2 + z^2 = 1,$$

over the flat five dimensional space R^5 with the associated metric

$$-du^2 - dv^2 + dx^2 + dy^2 + dz^2 = ds^2.$$

The Anti de Sitter spacetime describes a universe of negative Λ [3].

2 Causality implementation.

There are many ways in which people implement causality, in this work we will not fret over them, instead we are going to move the scenario on to special relativity where for every Lorentz frame the event B is in the future of event A if and only if B is in A 's light cone. Starting from the relativistic expression of the interval and using a flat metric with signature $(-, +, +, +)$:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2. \quad (2)$$

Where we are working in units of $c = h = 1$. Then the *mathematical* condition for a particle remains *inside* or *on* the light-cone is expressed as:

$$(ds)^2 \leq 0 \quad (3)$$

This means that a space-like or time-like particle can not have a speed higher than c . Inequation (3) just says that. But it also has some taste of causality. The problem is, perhaps, the fact that working with an inequation does not give further information about it. The question is: how to use (3) in order to manipulate causality?. We will propose an answer:

Since $(ds)^2$ is a quadratic expression and also $(ds)^2 \leq 0$, then it will be equal to another physical-meaning quadratic expression multiplied by a negative sign, this guarantees the correct sign we want for inequation (3) be satisfied.

Using (2) and (3), it would read something like this:

$$\begin{aligned} ds^2 &= -A^2 \\ -dt^2 + dx^2 + dy^2 + dz^2 &= -A^2, \end{aligned} \quad (4)$$

where A is a quantity to be determined from physics.

Now, another question arises now: what is the meaning of the quantity A ?. The answers is not so easy as before, but let's try to figure out something.

We should classified our world as something where we have *massive* and other *massless* objects like the protons and photons respectively. From Einstein's observations, no mass can live *on* the light-cone,

it must lie *inside* it!, i.e., the inequation (3) for a massless particle becomes in an equation. For our purposes, the quantity A *must be equal to zero in the massless case, otherwise, it must be different to zero*. Something between mass and the quantity A is happening, this can be achieved just saying that A is a function of an small massive test object dm , i.e. a differential of mass:

$$A = A(dm), \quad (5)$$

but, why nature can made a cumbersome function?. Then, let's try the easiest one, i.e., a linear function:

$$A = a_1 dm + a_0, \quad (6)$$

where a_1 and a_0 are the two constants needed to define a linear function. Using the ideas described above, we conclude that $a_0 = 0$. Then let's work out the a_1 case.

Since we are dealing with physics and a constant value, we should take the fundamental constants of physics in our analyze: the Newton's constant G , the charge of an electron e , the speed of light c and the Planck's constant h . From (4), using dimensional analysis and the Newton's law of gravitation, we can argue that G , the gravitation constant, is a good candidate to be the proportional quantity a_1 . This means that equation (4) turns to:

$$\begin{aligned} A &= Gdm \\ -dt^2 + dx^2 + dy^2 + dz^2 &= -G^2 dm^2 \end{aligned} \quad (7)$$

The last equation is the equation of an hyper-hyperboloid as it is shown in figure 1 We will call this hyper-hyperboloid as a *causal-hyperboloid*. This is not a hoax, it is just a beggarly tentative to approximate causality. We can also think that (7) shows another dimension associated to the mass of the particle.

Now, we can emphasize the following observations:

- Absolute past and absolute future are “connected” via the mass of the particle (see figure 1).
- Since the value of G is very small, the causal-hyperboloid was worked as the light-cone. This case turns back to Einstein's Special Relativity.

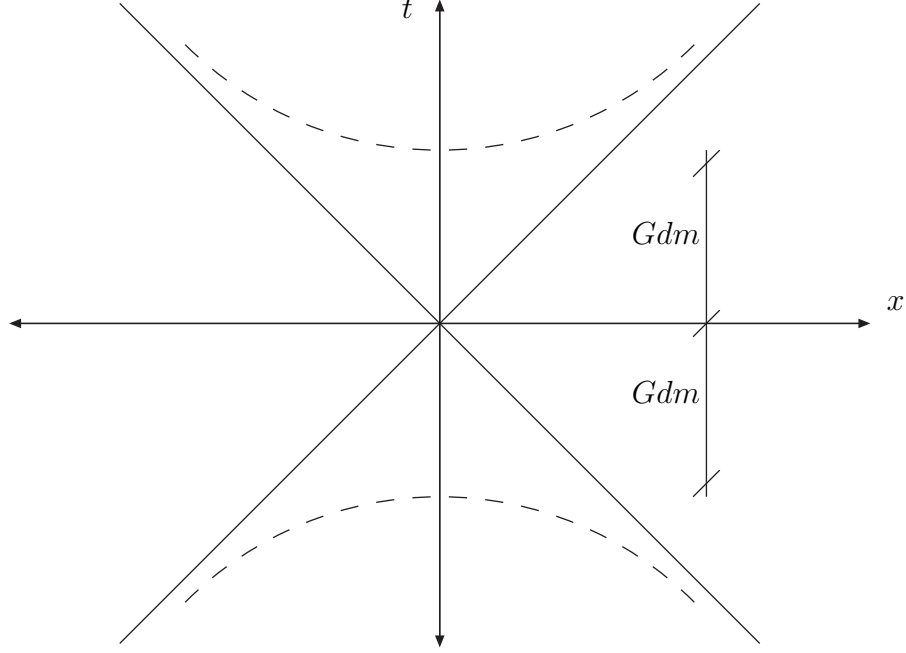


Figure 1: Light-cone and causal-hyperboloid.

- Equation (7) can be written as:

$$-dt^2 + dx^2 + dy^2 + dz^2 + G^2 dm^2 \doteq d\mathcal{S}^2, \quad (8)$$

where we have been introduced an “new element” $d\mathcal{S}^2$. In this way, equation (8) defines also a de Sitter space, as will be see in the next lines.

To improve a better analyze of (8), we will write it as

$$-dt^2 + e^{2\beta t} \cdot (dx^2 + dy^2 + dz^2 + G^2 dm^2) \doteq d\mathcal{S}^2,$$

using

$$\begin{aligned} \beta t &= \ln \beta \frac{v+w}{\alpha}, \\ \beta x &= \frac{x'}{v+w}, \\ \beta y &= \frac{y'}{v+w}, \end{aligned}$$

$$\begin{aligned}\beta z &= \frac{z'}{v+w}, \\ \beta m &= \frac{m'}{v+w},\end{aligned}$$

the metric goes to

$$-dv^2 + dw^2 + dx'^2 + dy'^2 + dz'^2 + G^2 dm'^2 = dS^2.$$

It is this last equation we would like to focus on, since it was derived from our approach of causality implementation, that we have now a de Sitter spacetime, see equation (1), but now in five dimensions (dS^5) [4, 3].

3 A comment on spacetime dimensions.

As we have seen in the previous section, the implementation of causality implies that we have, at least as an approximation, a de Sitter spacetime instead of the Minkowski space, but now with five dimensions, i.e., we are dealing with a dS^5 space, and this is a universe of positive cosmological constant Λ . On the other hand an Anti de Sitter space in five dimensions, AdS^5 , describes a universe of negative Λ and has as interesting applications in efforts to obtain a better understanding of string theory [5]. On the way of superstring theory, one problem consists that the spacetime now has ten dimensions. The idea is to use the compactification mechanism so finally we have a Minkowski M^4 spacetime and a compactified manifold \mathcal{R}^6 as can be seen in figure 2. From the considerations given above, now we have a de Sitter spacetime. Then we can fill in the rest of the space with another \mathcal{R}^5 space but if we really wish to get something symmetric, the choice is to pick up the Anti de Sitter spacetime. working in this manner, we should have a kind of symmetry in spacetime dimensions, in one hand a de Sitter spacetime with positive Λ and on the other hand a Anti de Sitter spacetime with negative Λ .

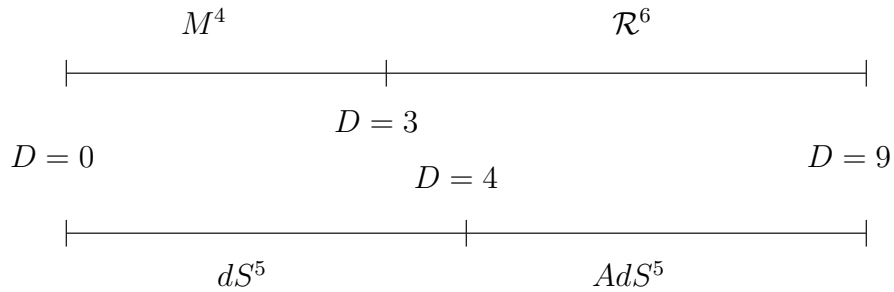


Figure 2: Spacetime distribution. D is the dimension and runs from 0 to 9.

4 Conclusions.

In this work we just intent to implement causality and observe how this relates the de Sitter spacetime in five dimensions. The idea of more dimensions is not a newer one but an older [6] and the pioneers were Th. Kaluza and O. Klein.

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